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Vertex operators in IIB matrix model

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Abstract: The vertex operators for the supergravity multiplet can be constructed through the Wilson lines in IIB matrix model. We investigate the structure of the vertex operators and the symmetry of their correlation functions. For this purpose, we perturb the theory by the Wilson lines dual to the supergravity multiplet. The structure of the Wilson lines can be determined by requiring $\mathcal{N} = 2$ SUSY under the low energy approximation. We argue that the generating functional of the correlators is invariant under local SUSY transformations of IIB supergravity.

Keywords: Superstrings and Heterotic Strings, M(atrix) Theories, Supergravity Models.

Contents

1. Introduction

One of the important problems in matrix model formulations of superstring theory is to identify gauge symmetries which are expected in closed string theory [1, 2]. In particular, we must identify local symmetries of supergravity. In the case of IIB matrix model, we expect to find local supersymmetry, local Lorentz, general coordinate transformation and three other gauge symmetries. Since the rest follows from local supersymmetry, it is expected to shed light on other gauge symmetries as well.

We recall IIB matrix model action [2]:

$$S = -\frac{1}{g^2}\,\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right), \tag{1.1}$$

here ψ is a ten-dimensional Majorana-Weyl spinor field, and A_μ and ψ are $N \times N$ hermitean matrices.[1] Since this model is formulated in terms of hermitean matrices, it has $\mathrm{U}(N)$ gauge symmetry. It has been postulated that the Wilson loops are the vertex operators for fundamental strings since they are the only gauge invariant observables [3]. They appear to play a key role to uncover local symmetries in matrix models.

Non-commutative (NC) gauge theory can be naturally obtained from matrix models by postulating non-commutative space-time [4, 5, 6]. The gauge invariant observables in NC gauge theory, Wilson lines, have been obtained from the Wilson loops in matrix models [7]. The Wilson lines share analogous properties with string theory such as the UV-IR mixing [11] and universal high energy behavior [12, 13, 14]. In string theory approach to NC gauge theory [10], the Wilson lines couple NC gauge fields to closed string modes [15]–[19]. In bosonic string theory context, we can understand gauge symmetries of closed string modes from the reparametrization invariance of the Wilson lines [20]. We are hence motivated to investigate analogous problem in superstring context. However the scope of this paper is limited to massless modes of superstring, namely the supergravity multiplet. Although we investigate a matrix model in this paper, it is straightforward to extend our results to NC gauge theory.

[1] We use the conventions $\{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}$ where $\eta = \mathrm{diag}(- + \ldots +)$.

The IIB matrix model possesses $\mathcal{N}=2$ supersymmetry as

$$\delta^{(1)}\psi = \frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\epsilon^1\,, \qquad \delta^{(1)}A_\mu = i\bar{\epsilon}^1\Gamma_\mu\psi\,, \tag{1.2}$$

and

$$\delta^{(2)}\psi = -\epsilon^2\,, \qquad \delta^{(2)}A_\mu = 0\,. \tag{1.3}$$

Up to the gauge symmetry and the equation of motion for ψ, we have the following commutation relations:

$$\left(\delta^{(1)}_{\epsilon_1}\delta^{(1)}_{\epsilon_2} - \delta^{(1)}_{\epsilon_2}\delta^{(1)}_{\epsilon_1}\right)\psi = 0\,, \qquad \left(\delta^{(1)}_{\epsilon_1}\delta^{(1)}_{\epsilon_2} - \delta^{(1)}_{\epsilon_2}\delta^{(1)}_{\epsilon_1}\right)A_\mu = 0\,. \tag{1.4}$$

We can also easily check the following commutators:

$$\left(\delta^{(1)}_{\epsilon}\delta^{(2)}_{\xi} - \delta^{(2)}_{\xi}\delta^{(1)}_{\epsilon}\right)\psi = 0\,, \qquad \left(\delta^{(1)}_{\epsilon}\delta^{(2)}_{\xi} - \delta^{(2)}_{\xi}\delta^{(1)}_{\epsilon}\right)A_\mu = i\bar{\epsilon}\Gamma_\mu\xi\,, \tag{1.5}$$

and

$$\left(\delta^{(2)}_{\xi_1}\delta^{(2)}_{\xi_2} - \delta^{(2)}_{\xi_2}\delta^{(2)}_{\xi_1}\right)\psi = 0\,, \qquad \left(\delta^{(2)}_{\xi_1}\delta^{(2)}_{\xi_2} - \delta^{(2)}_{\xi_2}\delta^{(2)}_{\xi_1}\right)A_\mu = 0\,. \tag{1.6}$$

If we take a linear combination of $\delta^{(1)}$ and $\delta^{(2)}$ as

$$\tilde{\delta}^{(1)} = \delta^{(1)} + \delta^{(2)}\,, \qquad \tilde{\delta}^{(2)} = \frac{1}{i}\left(\delta^{(1)} - \delta^{(2)}\right), \tag{1.7}$$

we obtain the $N=2$ supersymmetry algebra,

$$\begin{aligned}
\left(\tilde{\delta}^{(1)}_1\tilde{\delta}^{(1)}_2 - \tilde{\delta}^{(1)}_2\tilde{\delta}^{(1)}_1\right)\psi &= 0\,, \qquad \left(\tilde{\delta}^{(1)}_1\tilde{\delta}^{(1)}_2 - \tilde{\delta}^{(1)}_2\tilde{\delta}^{(1)}_1\right)A_\mu = i\bar{\epsilon}_1\Gamma_\mu\xi_2 + i\bar{\xi}_1\Gamma_\mu\epsilon_2\,,\\
\left(\tilde{\delta}^{(2)}_1\tilde{\delta}^{(2)}_2 - \tilde{\delta}^{(2)}_2\tilde{\delta}^{(2)}_1\right)\psi &= 0\,, \qquad \left(\tilde{\delta}^{(2)}_1\tilde{\delta}^{(2)}_2 - \tilde{\delta}^{(2)}_2\tilde{\delta}^{(2)}_1\right)A_\mu = i\bar{\epsilon}_1\Gamma_\mu\xi_2 + i\bar{\xi}_1\Gamma_\mu\epsilon_2\,,\\
\left(\tilde{\delta}^{(1)}_1\tilde{\delta}^{(2)}_2 - \tilde{\delta}^{(2)}_2\tilde{\delta}^{(1)}_1\right)\psi &= 0\,, \qquad \left(\tilde{\delta}^{(1)}_1\tilde{\delta}^{(2)}_2 - \tilde{\delta}^{(2)}_2\tilde{\delta}^{(1)}_1\right)A_\mu = 0\,.
\end{aligned} \tag{1.8}$$

These symmetry considerations force us to interpret the eigenvalues of A_μ as the space-time coordinates. We note here that ϵ and ξ can be regarded as the complex conjugate to each other.[2]

The IIB supergravity multiplet consists of a real graviton $h_{\mu\nu}$, a real fourth rank antisymmetric tensor $A_{\mu\nu\rho\sigma}$, a complex dilaton Φ, a complex dilatino λ, a complex antisymmetric tensor $B_{\mu\nu}$ and a complex gravitino η_μ. These fields may be introduced in IIB matrix model as the sources which couple to the Wilson lines. They transform in a definite way under local supersymmetry transformation in IIB supergravity [23]. The local supersymmetry is realized in the matrix model if the perturbed theory is invariant under such a transformation. We show that it is the case at the linearized level of the symmetry under a low energy approximation. This conclusion follows from $\mathcal{N}=2$ supersymmetry of IIB matrix model.

In section 2, we briefly recall current understandings of the Wilson lines in matrix models and string theory. In section 3, we investigate the vertex operators for the supergravity multiplet and the symmetries of their correlators. We conclude in section 4 with discussions.

[2] Such an interpretation is consistent with the fact that IIB matrix model can be related to Green-Schwarz action [21] in the Schild form [22] after the analytic continuation in the fermionic variables [2].

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2. Wilson lines in matrix models and string theory

In matrix models, a generic Wilson line is defined along an open contour C:

$$\begin{aligned} w(C) &= \mathrm{Str}\left[\prod_i \hat{O}_i v(C)\right], \\ v(C) &= P\exp\left\{i\int_C d\sigma (k(\sigma)^\mu A_\mu)\right\}. \end{aligned} \tag{2.1}$$

Here $k(\sigma)^\mu$ denotes the momentum density distributed along C. Since P denotes the path ordering, $\mathrm{Tr}\, v(C)$ is obtained from a familiar Wilson loop operator by assuming the gauge fields to be constant. $\hat{O}_i$ denotes matrices such as ψ, $[A_\mu, A_\nu]$, $[A_\mu, \psi]$ or products of them. The translation invariance requires that A_μ must appear through the commutators in $\hat{O}_i$. The symbol Str denotes the symmetric trace which specifies the ordering of the matrices. In Str, $\hat{O}_i$ should be treated as a single entity. Therefore Str amounts to average all possible ways of insertions of $\hat{O}_i$ into $v(C)$. Unlike the Wilson loops in gauge theory, the gauge invariant observables are specified by generic open contours.

Supergravity type long range interactions emerge in matrix models after integrating out off-diagonal components of the matrices. The Wilson lines appear in such an effective action. We recall the one-loop amplitude of IIB matrix model.

$$\begin{aligned} W =& -\mathcal{T}r\left(\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\nu\lambda}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\rho\mu}\right) - 2\mathcal{T}r\left(\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\mu\rho}\frac{1}{P^2}F_{\lambda\nu}\right) + \\ &+\frac{1}{2}\mathcal{T}r\left(\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\lambda\rho}\right) + \frac{1}{4}\mathcal{T}r\left(\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\right) + \\ &+ O((F_{\mu\nu})^5)\,. \end{aligned} \tag{2.2}$$

Here P_μ and $F_{\mu\nu}$ are operators acting on the space of matrices as

$$\begin{aligned} P_\mu X &= [p_\mu, X]\,, \\ F_{\mu\nu} X &= [f_{\mu\nu}, X]\,, \end{aligned} \tag{2.3}$$

where $f_{\mu\nu} = i[p_\mu, p_\nu]$. In matrix models, long-range interactions may be investigated by considering the background p_μ to be of the block-diagonal form. We can then decompose $f_{\mu\nu} = f^i_{\mu\nu} + f^j_{\mu\nu}$.

The long range interactions between well separated and localized gauge configurations $f^i_{\mu\nu}$ and $f^j_{\mu\nu}$ are

$$\begin{aligned} &-\frac{3}{r^8}\left(\mathrm{Str}_i\left[f_{\mu\nu}f_{\nu\rho}f_{\rho\sigma}f_{\sigma\mu} - \frac{1}{4}f_{\mu\nu}f_{\nu\mu}f_{\rho\sigma}f_{\sigma\rho}\right]\times \mathrm{Str}_j[1] + (i\leftrightarrow j)\right) - \\ &-\frac{12}{r^8}\left(\mathrm{Str}_i\left[f_{\nu\rho}f_{\rho\sigma}f_{\sigma\mu} - \frac{1}{4}f_{\nu\mu}f_{\rho\sigma}f_{\sigma\rho}\right]\times \mathrm{Str}_j[f_{\mu\nu}] + (i\leftrightarrow j)\right) - \\ &-\frac{12}{r^8}\mathrm{Str}_i[f_{\mu\nu}f_{\nu\rho}]\,\mathrm{Str}_j[f_{\mu\sigma}f_{\sigma\rho}] + \frac{3}{2r^8}\mathrm{Str}_i[f_{\mu\nu}f_{\nu\mu}]\,\mathrm{Str}_j[f_{\rho\sigma}f_{\sigma\rho}] + \\ &+\frac{9}{r^8}\mathrm{Str}_i[f_{[\mu\nu}f_{\rho\sigma]}]\,\mathrm{Str}_j[f_{[\mu\nu}f_{\rho\sigma]}]\,, \end{aligned} \tag{2.4}$$

where Str_i denotes the symmetric trace over the i-th sub-matrix space. $[\mu\nu\rho\sigma]$ denotes the anti-symmetrization among the indices with unit weight. We use $(\mu\nu)$ for the symmetrization among the indices later. The first line in the above expression can be understood by the exchange of a complex dilaton field, the second by a complex second rank antisymmetric tensor field, the third by a graviton and the last by a fourth rank antisymmetric tensor, respectively. Here we have not addressed the issue of separating a dilaton from the trace part of a graviton. In (2.4), we note that the Wilson loops act as the vertex operators. We can also read-off the bosonic structure of the relevant vertex operators. We indeed find the identical structure for the vertex operators in the following section. This kind of investigation has been extended with the inclusion of fermionic backgrounds in [24, 25, 26].

In NC gauge theory context, such a phenomena is called as the UV-IR mixing [9, 11]. It can be uniquely traced to the non-planar sector. In NC gauge theory, the Wilson lines appear in the one-loop effective action through non-planar diagrams. In fully NC gauge theory, eq. (2.2) can be evaluated as

$$\begin{aligned} W = V \int \frac{d^D k}{(2\pi)^D} \mathrm{Tr} \Bigg[& \exp(-ik\cdot\hat{x}) \times \\ & \times \Bigg\{ -\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\nu\lambda}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\rho\mu} - \\ & - 2\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\mu\rho}\frac{1}{P^2}F_{\lambda\nu} + \\ & + \frac{1}{2}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\lambda\rho} + \\ & + \frac{1}{4}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho}\frac{1}{P^2}F_{\mu\nu}\frac{1}{P^2}F_{\lambda\rho} \Bigg\} \exp(ik\cdot\hat{x}) \Bigg] + \\ + O((F_{\mu\nu})^5)\,, \end{aligned} \tag{2.5}$$

where we normalize $\mathrm{Tr}[\hat{1}] = 1$. We assume $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$ and the rank of θ is D. For small external momenta, we can evaluate the above as

$$\begin{aligned} V\theta^{8-D} \int \frac{d^D q}{(2\pi)^D} \int d^D y \exp(iq\cdot y) \times & \\ \times \Bigg\{ -\frac{24}{|y|^8} \mathrm{Tr}\left[\exp(-iq\cdot\hat{x}) \left(f_{\nu\rho}f_{\rho\sigma}f_{\sigma\mu} - \frac{1}{4}f_{\nu\mu}f_{\rho\sigma}f_{\sigma\rho}\right)\right] \mathrm{Tr}[\exp(iq\cdot\hat{x})f_{\mu\nu}] - & \\ - \frac{12}{|y|^8}\mathrm{Tr}[\exp(-iq\cdot\hat{x})f_{\mu\nu}f_{\nu\rho}]\,\mathrm{Tr}[\exp(iq\cdot\hat{x})f_{\mu\sigma}f_{\sigma\rho}] + & \\ + \frac{3}{2|y|^8}\mathrm{Tr}[\exp(-iq\cdot\hat{x})f_{\mu\nu}f_{\nu\mu}]\,\mathrm{Tr}[\exp(iq\cdot\hat{x})f_{\rho\sigma}f_{\sigma\rho}] + & \\ + \frac{9}{|y|^8}\mathrm{Tr}\left[\exp(-iq\cdot\hat{x})f_{[\mu\nu}f_{\rho\sigma]}\right]\mathrm{Tr}\left[\exp(iq\cdot\hat{x})f_{[\mu\nu}f_{\rho\sigma]}\right] \Bigg\}\,. & \end{aligned} \tag{2.6}$$

Here we can indeed recognize the Wilson lines which are expanded by NC gauge fields to the leading non-trivial order. Various aspects of the Wilson lines in NC field theory have been studied in [29, 30].

The structure of the Wilson line operators has been also investigated in superstring theory [17]. We can evaluate the one point functions of the vertex operators for the supergravity multiplet with gauge fields at the boundary of the disk. The correlation function which we need to evaluate is

$$(z-\bar{z})^2 \left\langle V^{(-1,-1)}(z)\,\mathrm{Tr}\,P\exp\left(i\int dt U^{(0)}(t)\right)\right\rangle . \tag{2.7}$$

$U^{(0)}(t)$ is the operator in the 0-picture for open string

$$U^{(0)}(t) = i\Phi_\mu \partial_\perp X^\mu - i[\Phi_\mu,\Phi_\nu]\Psi^\mu\Psi^\nu\,, \tag{2.8}$$

where Φ_μ are $N\times N$ hermitean matrices.

Let us consider the vertex operator for a graviton and an antisymmetric tensor in the $(-1,-1)$-picture

$$V^{(-1,-1)}(z) = \delta(\gamma)\delta(\bar{\gamma})\psi^\mu(z)\bar{\psi}^\nu(\bar{z})e^{ik\cdot X(z)}\,. \tag{2.9}$$

In the appropriate scaling limit, we find

$$\begin{aligned}(z-\bar{z})&\left\langle \psi^\mu(z)\bar{\psi}^\nu(\bar{z})e^{ik\cdot X(z)}\,\mathrm{Tr}\,P\exp\left(i\int dt U^{(0)}(t)\right)\right\rangle = \\ &= \mathrm{Tr}\left[P\exp\left(i\int_0^1 d\tau k\cdot A\right)\times\right.\\ &\qquad \left.\times\left(\frac{i}{2\pi}\int_0^1 d\tau_1[A^\mu,A^\nu] + \frac{1}{(2\pi)^2\alpha'}\int_0^1 d\tau_1[A^\mu,A^\rho]\int_0^1 d\tau_2[A_\rho,A^\nu]\right)\right],\end{aligned} \tag{2.10}$$

where $A^\mu = 2\pi\alpha'\Phi^\mu$. In this way we can determine the bosonic part of the vertex operators for $B_{\mu\nu}$ and $h_{\mu\nu}$. The results are consistent with the one-loop effective action of NC gauge theory.

Let us consider the simplest gauge invariant operator which corresponds to a straight Wilson line operator in NC gauge theory:

$$\mathrm{Tr}[\exp(ik\cdot A)]\,. \tag{2.11}$$

In order to understand the SUSY multiplet to which it belongs, we consider the SUSY transformation eq. (1.2). We observe that the SUSY transformation $\delta^{(1)}$ whose generator satisfies $k_\mu\Gamma^\mu\epsilon = 0$ commutes with the Wilson line. If k is a null vector, the straight Wilson line is a BPS state since it preserves the half of SUSY. Let us choose the Lorentz frame such that $k_\mu\Gamma^\mu = k^+\Gamma^-$. The broken SUSY generators satisfy $\Gamma^+\lambda = 0$. The massless SUSY multiplets are generated by such broken SUSY generators $\{\lambda^a\}$ from the null Wilson line. They can be represented by a superfield Ψ which is a polynomial in λ^a up to the eighth order.

$$\begin{aligned}\Psi &= A + \psi^a\lambda^a + \frac{1}{2}A^{ab}\lambda^a\lambda^b - \\ &\quad - \frac{1}{3!}\psi^{abc}\lambda^a\lambda^b\lambda^c + \frac{1}{4!}A^{abcd}\lambda^a\lambda^b\lambda^c\lambda^d + \\ &\quad + \frac{1}{3!\cdot 5!}\psi^{abc*}\epsilon^{a\ldots h}\lambda^d\cdots\lambda^h - \frac{1}{2\cdot 6!}A^{ab*}\epsilon^{a\ldots h}\lambda^c\cdots\lambda^h + \\ &\quad + \frac{1}{7!}\psi^{a*}\epsilon^{a\ldots h}\lambda^b\cdots\lambda^h + \frac{1}{8!}A^*\epsilon^{a\ldots h}\lambda^a\cdots\lambda^h\,.\end{aligned} \tag{2.12}$$

It is in fact known that these 2^8 fields form the IIB supergravity multiplet [27]. The light-cone gauge formulation of Wilson loops can be extended to massive states in IIB matrix model [28].

The merit of this light-cone type argument is its robustness against possible quantum corrections due to the BPS nature of the multiplet. The drawback is that it is oblivious to gauge symmetries. We may draw an analogy with U(1) gauge theory here. The on-shell photon has two transverse degrees of freedom. Its covariant off-shell extension is described by U(1) gauge theory. It is most likely that the covariant off-shell extension of the straight Wilson lines and their descendants is described by supergravity. In the following section, we identify such expected local symmetries.

3. $N = 2$ SUSY and the vertex operators

In this section, we construct the vertex operators in IIB matrix model which couple to the supergravity multiplet. The relevant operators can be constructed through the Wilson lines. We start with the simplest operator which corresponds to a straight Wilson line in NC gauge theory:

$$\mathrm{Tr}[\exp(ik\cdot A)]\Phi(k)\,. \tag{3.1}$$

In bosonic string theory, $\Phi(k)$ has been interpreted as a tachyon field with momentum k. A natural question is how to interpret this operator in superstring. In this section we argue that Φ can be interpreted as a dilaton field in the IIB supergravity multiplet.

Our strategy is to consider the generating functional of the Wilson line correlators:

$$e^{W(\Phi_i)} = \left\langle e^{V_i\Phi_i}\right\rangle, \tag{3.2}$$

where the average is taken with respect to the IIB matrix model action given in eq. (1.1). Φ_i and V_i denote the fields and dual vertex operators descended from $\Phi(k)$ and $\mathrm{Tr}[\exp(ik\cdot A)]$ by $\mathcal{N} = 2$ SUSY. We will shortly find that they form the IIB supergravity multiplet as expected. By considering the change of variables which is identical to SUSY transformation in (1.2) and (1.3), we can derive the Ward identities in a standard way:

$$\left\langle e^{V_i\Phi_i}\right\rangle = \left\langle e^{V_i(A+\delta A,\psi+\delta\psi)\Phi_i}\right\rangle. \tag{3.3}$$

Our task is to construct the vertex operators $\{V_i\}$ in order to satisfy the following relation with the judicious choice of $\delta\Phi_i$[3]

$$\left\langle e^{V_i(A+\delta A,\psi+\delta\psi)\Phi_i}\right\rangle = \left\langle e^{V_i(\Phi_i+\tilde{\delta}\Phi_i)}\right\rangle. \tag{3.4}$$

If it is successful, we can derive the symmetry of the generating functional for the correlators of Wilson lines:

$$e^{W(\Phi_i)} = e^{W(\Phi_i+\tilde{\delta}\Phi_i)}. \tag{3.5}$$

Since our scope is limited to the supergravity multiplet in this paper, we carry out this program under a low energy approximation. We find that the symmetry of $W(\Phi_i)$ coincides with the local SUSY transformation of IIB supergravity at the linearized level [23]. We denote it as SUGRA transformation in what follows.

[3]This strategy has been employed in several other contexts. See [31] for a Matrix theory application.

With these motivations, let us apply SUSY transformation to eq. (3.1):

$$\delta\,\mathrm{Tr}[\exp(ik\cdot A)]\Phi(k) = \mathrm{Tr}[\exp(ik\cdot A)\bar{\psi}]k\cdot\Gamma\epsilon^1\Phi(k)\,. \tag{3.6}$$

In order to satisfy eq. (3.4), we are led to add a new operator to eq. (3.1) which contains the dilatino field λ:

$$\begin{aligned} &-\mathrm{Tr}[\exp(ik\cdot A)\bar{\psi}]\lambda\,,\\ &\tilde{\delta}\lambda = -k\cdot\Gamma\epsilon^1\Phi(k)\,. \end{aligned} \tag{3.7}$$

Through eq. (3.5), we have identified SUGRA transformation of the dilatino field as (3.7).

We in turn need to consider SUSY transformation of the newly introduced vertex operator:

$$\begin{aligned} -\delta\,\mathrm{Tr}[\exp(ik\cdot A)\bar{\psi}]\lambda = {}& \mathrm{Tr}[\exp(ik\cdot A)]\bar{\epsilon^2}\lambda + \mathrm{Tr}\left[\exp(ik\cdot A)\frac{i}{2}[A_\mu,A_\nu]\right]\bar{\epsilon^1}\Gamma^{\mu\nu}\lambda - \\ & - \mathrm{Str}\left[\exp(ik\cdot A)\bar{\psi}k\cdot\Gamma\epsilon^1\bar{\psi}\lambda\right]. \end{aligned} \tag{3.8}$$

The symmetric trace, Str implies the following operation

$$\begin{aligned} &\mathrm{Str}[\exp(ik\cdot A)\psi_\alpha\psi_\beta] = \\ &\quad = \int_0^1 d\sigma\int_\sigma^1 d\sigma'\,\mathrm{Tr}\left[\exp(i\sigma k\cdot A))\psi_\alpha\exp(i(\sigma'-\sigma)k\cdot A))\psi_\beta\exp(i(1-\sigma')k\cdot A))\right] - \\ &\qquad - (\alpha\leftrightarrow\beta)\,, \end{aligned} \tag{3.9}$$

where the $-$ sign is due to the fermionic nature of ψ. Starting from the simply traced object in eq. (3.1), we only obtain symmetric traced objects. We can further recombine fermionic variables through the following Fierz identity

$$\bar{\phi}_\beta\psi_\alpha = -\frac{1}{16}\Gamma^\mu_{\alpha\beta}\bar{\phi}\Gamma_\mu\psi + \frac{1}{16\cdot 3!}\Gamma^{\mu\nu\rho}_{\alpha\beta}\bar{\phi}\Gamma_{\mu\nu\rho}\psi - \frac{1}{16\cdot 5!}\Gamma^{\mu\nu\rho\sigma\tau}_{\alpha\beta}\bar{\phi}\Gamma_{\mu\nu\rho\sigma\tau}\psi\,, \tag{3.10}$$

where ψ and ϕ are Majorana-Weyl spinors in ten dimensions.

The first term in eq. (3.8) can be interpreted as the SUGRA transformation of a dilaton:

$$\tilde{\delta}\Phi = \bar{\epsilon^2}\lambda\,. \tag{3.11}$$

The remaining terms lead us to introduce the coupling with the antisymmetric tensor field $B^{\mu\nu}$ in addition

$$\begin{aligned} &\mathrm{Tr}\exp(ik\cdot A)\frac{i}{2}[A_\mu,A_\nu]B^{\mu\nu} - \frac{i}{48}\,\mathrm{Str}\exp(ik\cdot A)\bar{\psi}\Gamma_{\mu\nu\rho}\psi H^{\mu\nu\rho}\,,\\ &\tilde{\delta}B^{\mu\nu} = \bar{\epsilon^1}\Gamma^{\mu\nu}\lambda\,,\\ &H_{\mu\nu\rho} \equiv \partial_\rho B_{\mu\nu} + \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu}\,. \end{aligned} \tag{3.12}$$

Here only the anti-symmetric part in the fermion bilinear term contributes due to the definition of Str as in eq. (3.9). The application of ∂_μ should be understood as the multiplication by ik_μ in this paper. We have also assumed that $\Gamma^\mu\partial_\mu\lambda = 0$. We note that the vertex operator in eq. (3.12) can be shown to be invariant under the following gauge transformation of $B_{\mu\nu}$ field

$$\tilde{\delta} B_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]}\,. \tag{3.13}$$

The SUSY transformation of the vertex operator in eq. (3.12) gives rise to

$$\begin{aligned}
&\frac{i}{24}\,\mathrm{Tr}\exp(ik\cdot A)\bar{\epsilon^2}\Gamma_{\mu\nu\rho}\psi H^{\mu\nu\rho} + \\
&\quad + \frac{1}{48}\,\mathrm{Str}\exp(ik\cdot A)\bar{\psi}[A_\alpha, A_\beta]\Gamma^\alpha(\Gamma^{\beta\nu\rho\lambda}H_{\nu\rho\lambda} + 9\Gamma^{\rho\lambda}H_{\beta\rho\lambda})\epsilon^1 - \\
&\quad - \frac{i}{48}\,\mathrm{Str}\exp(ik\cdot A)\bar{\psi}k\cdot\Gamma\epsilon^1\bar{\psi}\Gamma_{\mu\nu\rho}\psi H^{\mu\nu\rho}\,.
\end{aligned} \tag{3.14}$$

The first term of eq. (3.14) can be obtained from eq. (3.7) by postulating the following SUGRA transformation for dilatino field:

$$\tilde{\delta}\lambda = -\frac{i}{24}\Gamma_{\mu\nu\rho}\epsilon^2 H^{\mu\nu\rho}\,. \tag{3.15}$$

The second term necessitates us to introduce the following vertex operator which couples to gravitino field η_μ:

$$\begin{aligned}
&\mathrm{Str}\exp(ik\cdot A)\bar{\psi}[A_\alpha, A_\beta]\Gamma^\alpha\eta^\beta\,, \\
&\tilde{\delta}\eta^\beta = \frac{1}{48}(\Gamma^{\beta\nu\rho\lambda}H_{\nu\rho\lambda} + 9\Gamma_{\rho\lambda}H^{\beta\rho\lambda})\epsilon^1\,.
\end{aligned} \tag{3.16}$$

We can show that this coupling is invariant under local SUSY transformation $\tilde{\delta}\eta_\mu = -ik_\mu\epsilon^2$ by using the equations of motion for ψ.

We are left with the third term in (3.14). It is O(k) in comparison to the rest and hence we may argue that it is irrelevant in the low energy limit. It might be explained by introducing a massive fermionic mode $\Psi^{[\mu\nu]}$ which is expected at the first excited level in superstring as follows

$$\begin{aligned}
&\frac{1}{16}\,\mathrm{Str}\exp(ik\cdot A)\bar{\psi}\Gamma_{\mu\nu\rho}\psi\bar{\psi}\partial^{[\rho}\Psi^{\mu\nu]}\,, \\
&\tilde{\delta}\Psi^{[\mu\nu]} = -ik\cdot\Gamma B^{\mu\nu}\epsilon^1\,.
\end{aligned} \tag{3.17}$$

Although it is a very interesting problem to understand the vertex operators for massive modes, it is beyond the scope of this paper. We are content to postpone this question by arguing that it is irrelevant in the low energy approximation.

The SUSY transformation of the gravitino coupling in eq. (3.16) gives rise to:

$$\begin{aligned}
&-\,\mathrm{Tr}\exp(ik\cdot A)[A_\alpha, A_\beta]\bar{\epsilon^2}\Gamma^\alpha\eta^\beta + \\
&+ i\,\mathrm{Str}\exp(ik\cdot A)[A_\alpha, A_\beta][A^\beta, A_\mu]\bar{\epsilon^1}\Gamma^\alpha\eta^\mu - \\
&- \frac{i}{2}\,\mathrm{Str}\exp(ik\cdot A)[A_\alpha, A_\beta][A_\gamma, A_\mu]\bar{\epsilon^1}\Gamma^{\alpha\beta\gamma}\eta^\mu + \\
&+ \frac{i}{2}\,\mathrm{Str}\exp(ik\cdot A)\bar{\psi}\Gamma_\alpha[\psi, A_\mu]\bar{\epsilon^1}\Gamma^\alpha\eta^\mu + \\
&+ \frac{1}{48}\,\mathrm{Str}\exp(ik\cdot A)\bar{\psi}\Gamma_{\alpha\beta\gamma}\psi[A_\mu, k\cdot A]\bar{\epsilon^1}\Gamma^{\alpha\beta\gamma}\eta^\mu + \\
&+ \mathrm{Str}\exp(ik\cdot A)\bar{\epsilon^1}\Gamma_\rho\psi[A_\alpha, A_\beta]\bar{\psi}(k^\beta\Gamma^\alpha\eta^\rho - k^\rho\Gamma^\alpha\eta^\beta)\,.
\end{aligned} \tag{3.18}$$

The first term in eq. (3.18) can be obtained from eq. (3.12) by assuming the following transformation for $B_{\mu\nu}$ field

$$\tilde{\delta} B_{\mu\nu} = 2i\bar{\epsilon}^2 \Gamma_{[\mu}\eta_{\nu]} \,. \tag{3.19}$$

The terms in the last two lines in eq. (3.18) are $\mathrm{O}(k)$. We also note that the antisymmetric part of the following term is also $\mathrm{O}(k)$ since

$$\begin{aligned} \frac{i}{4} \,\mathrm{Str} \exp(ik \cdot A) \bar{\psi} (\Gamma_\alpha [\psi, A_\mu] - \Gamma_\mu [\psi, A_\alpha]) \bar{\epsilon}^1 \Gamma^\alpha \eta^\mu = & \\ = \frac{i}{8} \,\mathrm{Str} \exp(ik \cdot A) \bar{\psi} (\Gamma^\beta \Gamma_{\alpha\mu} - \Gamma_{\alpha\mu} \Gamma^\beta) [\psi, A_\beta] \bar{\epsilon}^1 \Gamma^\alpha \eta^\mu & \\ = \frac{i}{8} \,\mathrm{Str} \exp(ik \cdot A) \bar{\psi} \Gamma^{\beta\alpha\mu} \psi [A_\beta, ik \cdot A] \bar{\epsilon}^1 \Gamma_\alpha \eta_\mu \,, & \end{aligned} \tag{3.20}$$

where we have used the equation of motion.

In order to account for eq. (3.18) in terms of SUGRA transformations, we need to introduce the Wilson lines which couple to a graviton $h_{\alpha\mu}$ and a four-th rank antisymmetric field $A_{\mu\nu\rho\sigma}$:

$$\begin{aligned} & \mathrm{Str} \exp(ik \cdot A) \left([A^\alpha, A^\beta][A^\mu, A_\beta] + \frac{1}{2} \bar{\psi} \Gamma^{(\alpha} [A^{\mu)}, \psi] \right) h_{\alpha\mu} + \\ & \quad + \frac{1}{2} \,\mathrm{Str} \exp(ik \cdot A) \bar{\psi} \Gamma^{\rho\beta(\alpha} \psi [A^{\mu)}, A_\beta] \partial_\rho h_{\alpha\mu} - \\ & \quad - \frac{i}{2} \,\mathrm{Str} \exp(ik \cdot A) [A^\alpha, A^\beta][A^\mu, A^\nu] A_{\alpha\beta\mu\nu} , \\ & \tilde{\delta} h_{\alpha\mu} = -i \bar{\epsilon}^1 \Gamma_{(\alpha} \eta_{\mu)} \,, \\ & \tilde{\delta} A_{\alpha\beta\mu\nu} = \bar{\epsilon}^1 \Gamma_{[\alpha\beta\mu} \eta_{\nu]} \,. \end{aligned} \tag{3.21}$$

We can show that the couplings in eq. (3.21) are invariant under the following gauge transformations after using the equation of motion

$$\tilde{\delta} h_{\alpha\mu} = \partial_\mu \xi_\alpha + \partial_\alpha \xi_\mu \,, \qquad \tilde{\delta} A_{\mu\nu\rho\sigma} = \partial_{[\mu} \Lambda_{\nu\rho\sigma]} \,. \tag{3.22}$$

We have fixed the $\mathrm{O}(k)$ term in eq. (3.21) in order to satisfy the invariance with respect to the general coordinate transformation. After this procedure, there remains the following discrepancy between SUGRA transformation of (3.21) and (3.18)

$$\begin{aligned} \mathrm{Str} \exp(ik \cdot A) \Bigg\{ & \frac{1}{48} \bar{\psi} \Gamma_{\alpha\beta\gamma} \psi [A_\mu, A^\rho] \bar{\epsilon}^1 k_\rho \Gamma^{\alpha\beta\gamma} \eta^\mu + \\ & + \bar{\epsilon}^1 k_\beta \Gamma_\rho \psi [A_\alpha, A^\beta] \bar{\psi} \Gamma^\alpha \eta^\rho - \bar{\epsilon}^1 k_\rho \Gamma^\rho \psi [A_\alpha, A_\beta] \bar{\psi} \Gamma^\alpha \eta^\beta - \\ & - \frac{1}{8} \bar{\psi} \Gamma^{\beta\alpha\mu} \psi [A_\beta, A^\rho] \bar{\epsilon}^1 k_\rho \Gamma_\alpha \eta_\mu - \frac{1}{2} \bar{\psi} \Gamma^{\rho\beta(\alpha} \psi [A^{\mu)}, A_\beta] \bar{\epsilon}^1 k_\rho \Gamma_\alpha \eta_\mu \Bigg\} . \end{aligned} \tag{3.23}$$

Although it is necessary to interpret these terms, it will be left to future investigations since we can ignore $\mathrm{O}(k)$ terms under the low energy approximation.

Although $h_{\mu\alpha}$ and $A_{\mu\nu\rho\sigma}$ are real fields, $\mathcal{N} = 2$ SUSY requires the introduction of the complex $B_{\mu\nu}$, dilaton and dilatino fields. We can in principle determine the structure of the Wilson lines dual to the complex conjugate of them by repeating this process. The SUSY transformation of eq. (3.21) is investigated in appendix A. Although we have shown that the result is $\mathrm{O}(k)$, our physical interpretation of it is still incomplete.

However we believe that the structure of the anti-gravitino vertex operator can be fixed by the requirement of gauge invariance. Here we make use of the fact that we can construct another conserved fermionic current in addition to that in (3.16). We propose to couple $\bar{\eta}^c_\mu$ to such a current as follows:

$$\begin{aligned}
-\frac{i}{2}\,\mathrm{Str}\exp(ik\cdot A)\bar{\eta}^c_\mu[A^\mu,A_\nu][A_\rho,A_\sigma]\Gamma^{\rho\sigma}\Gamma^\nu\psi &= \\
= -i\,\mathrm{Str}\exp(ik\cdot A)\bar{\eta}^c_\mu[A^\mu,A_\nu][A^\nu,A_\sigma]\Gamma^\sigma\psi &- \\
-\frac{i}{2}\,\mathrm{Str}\exp(ik\cdot A)\bar{\eta}^c_\mu[A^\mu,A_\nu][A_\rho,A_\sigma]\Gamma^{\nu\rho\sigma}\psi\,. &\qquad (3.24)
\end{aligned}$$

We can show the gauge invariance of the coupling corresponding to local SUSY $\delta\bar{\eta}^c_\mu = ik_\mu\bar{\epsilon}^1$ by using the equation of motion and neglecting cubic terms in ψ.

The SUSY transformation of eq. (3.24) leads to

$$\begin{aligned}
& i\,\mathrm{Str}\exp(ik\cdot A)\bar{\eta}^c_\mu[A^\mu,A_\nu][A^\nu,A_\sigma]\Gamma^\sigma\epsilon^2 + \\
&\quad +\frac{i}{2}\,\mathrm{Str}\exp(ik\cdot A)\bar{\eta}^c_\mu[A^\mu,A_\nu][A_\rho,A_\sigma]\Gamma^{\nu\rho\sigma}\epsilon^2 - \\
&\quad -2\,\mathrm{Str}\exp(ik\cdot A)[A^\mu,A_\nu][A^\nu,A_\sigma][A^\sigma,A_\alpha]\bar{\eta}^c_\mu\Gamma^\alpha\epsilon^1 + \\
&\quad +\frac{1}{2}\,\mathrm{Str}\exp(ik\cdot A)[A^\mu,A_\nu][A_\rho,A_\sigma][A^\sigma,A^\rho]\bar{\eta}^c_\mu\Gamma^\nu\epsilon^1\,, \qquad (3.25)
\end{aligned}$$

where we have neglected fermionic terms. It in turn implies

$$\begin{aligned}
\tilde{\delta}h_{\alpha\mu} &= -i\bar{\eta}^c_{(\mu}\Gamma_{\alpha)}\epsilon^2\,, \\
\tilde{\delta}A_{\alpha\beta\mu\nu} &= -\bar{\eta}^c_{[\alpha}\Gamma_{\beta\mu\nu]}\epsilon^2\,. \qquad (3.26)
\end{aligned}$$

We also need to introduce the coupling to $B^c_{\mu\nu}$ field:

$$\begin{aligned}
& -iB^c_{\mu\alpha}\,\mathrm{Str}\exp(ik\cdot A)\left([A^\mu,A_\nu][A^\nu,A_\sigma][A^\sigma,A^\alpha]-\frac{1}{4}[A^\mu,A^\alpha][A_\rho,A_\sigma][A^\sigma,A^\rho]\right), \\
& \tilde{\delta}B^c_{\mu\nu} = -2i\bar{\eta}^c_{[\mu}\Gamma_{\nu]}\epsilon^1\,. \qquad (3.27)
\end{aligned}$$

This coupling can be shown to be gauge invariant after using the equation of motion.

Since we have neglected fermionic terms in the $B^c_{\mu\nu}$ vertex operator, we can no longer determine the anti-dilatino vertex operator by SUSY transformations from it. In order to proceed further, we now resort to use consistency arguments. Firstly we expect to obtain the SUGRA transformations which are complex conjugate to the known type. Secondly we also expect to obtain the vertex operators which are consistent with the one-loop effective action in section 2. We may postulate the anti-dilatino coupling as follows since it satisfies these consistency requirements,

$$\begin{aligned}
& -i\bar{\lambda}^c\Gamma^{\mu\nu}\,\mathrm{Str}\exp(ik\cdot A)\psi\left([A_\mu,A_\rho][A^\rho,A_\sigma][A^\sigma,A_\nu]-\frac{1}{4}[A_\mu,A_\nu][A_\rho,A_\sigma][A^\sigma,A^\rho]\right) - \\
& \quad -\frac{i}{30}\bar{\lambda}^c\Gamma^{\mu\nu\rho\sigma\tau\lambda}\,\mathrm{Str}\exp(ik\cdot A)\psi[A_\mu,A_\nu][A_\rho,A_\sigma][A_\tau,A_\lambda]\,. \qquad (3.28)
\end{aligned}$$

With this ansatz, the SUSY transformation of the anti-dilatino coupling results in

$$
\begin{aligned}
& i\bar{\lambda}^c \Gamma^{\mu\nu} \epsilon^2 \,\mathrm{Str} \exp(ik\cdot A) \times \\
&\quad \times \left([A_\mu, A_\rho][A^\rho, A_\sigma][A^\sigma, A_\nu] - \frac{1}{4}[A_\mu, A_\nu][A_\rho, A_\sigma][A^\sigma, A^\rho]) \right) + \\
&\quad + \frac{i}{30} \bar{\lambda}^c \Gamma^{\mu\nu\rho\sigma\tau\lambda} \epsilon^2 \,\mathrm{Str} \exp(ik\cdot A)[A_\mu, A_\nu][A_\rho, A_\sigma][A_\tau, A_\lambda] + \\
&\quad + \bar{\lambda}^c \epsilon^1 \,\mathrm{Str} \exp(ik\cdot A) \times \\
&\quad \times \left([A_\mu, A_\nu][A^\nu, A_\sigma][A^\sigma, A_\alpha][A^\alpha, A^\mu] - \frac{1}{4}[A_\mu, A_\alpha][A^\alpha, A^\mu][A_\rho, A_\sigma][A^\sigma, A^\rho] \right) + \\
&\quad + \frac{1}{60} \bar{\lambda}^c \Gamma^{\mu\nu\rho\sigma\tau\lambda\alpha\beta} \epsilon^1 \,\mathrm{Str} \exp(ik\cdot A)[A_\alpha, A_\beta][A_\mu, A_\nu][A_\rho, A_\sigma][A_\tau, A_\lambda]\,. \qquad (3.29)
\end{aligned}
$$

Let us focus on ϵ^2 dependent part first. The second rank antisymmetric tensor part can be explained by the following SUGRA transformation

$$
\tilde{\delta} B^c_{\mu\nu} = -\bar{\lambda}^c \Gamma_{\mu\nu} \epsilon^2\,. \qquad (3.30)
$$

The sixth rank antisymmetric tensor part presumably requires the introduction of the sixth rank tensor field. We may interpret it as a massive mode since we expect such a mode at the first excited level in superstring. It possesses the required gauge symmetry to remove the negative norm states.

Let us move on to ϵ^1 dependent part. The scalar part requires us to include the anti-dilaton coupling as

$$
\begin{aligned}
& -\Phi^c \,\mathrm{Str} \exp(ik\cdot A) \Big([A_\mu, A_\nu][A^\nu, A_\sigma][A^\sigma, A_\alpha][A^\alpha, A^\mu] - \\
&\qquad\qquad - \frac{1}{4}[A_\mu, A_\alpha][A^\alpha, A^\mu][A_\rho, A_\sigma][A^\sigma, A^\rho] \Big)\,, \\
& \tilde{\delta}\Phi^c = -\bar{\lambda}^c \epsilon^1\,. \qquad (3.31)
\end{aligned}
$$

The eighth rank antisymmetric tensor part again may be dealt with by introducing a corresponding massive mode. It possesses the required gauge symmetry. Since we have exhausted the entire super gravity multiplet, this concludes our heuristic arguments to construct the vertex operators dual to the IIB supergravity multiplet.

Here we summarize the SUGRA transformations of the source fields which have been identified in this section. They are the symmetry of the generating functional of the Wilson line correlators, $W(\Phi_i)$. Admittedly the evidence becomes weaker for the lower members of the list since we can only offer consistency arguments for them.

$$
\begin{aligned}
\tilde{\delta}\Phi &= \bar{\epsilon^2}\lambda\,, \\
\tilde{\delta}\lambda &= i\Gamma^\mu \epsilon^1 \partial_\mu \Phi - \frac{i}{24}\Gamma_{\mu\nu\rho}\epsilon^2 H^{\mu\nu\rho}\,, \\
\tilde{\delta} B_{\mu\nu} &= \bar{\epsilon^1}\Gamma_{\mu\nu}\lambda + 2i\bar{\epsilon^2}\Gamma_{[\mu}\eta_{\nu]}\,, \\
\tilde{\delta}\eta_\mu &= \frac{1}{48}(\Gamma_{\mu\nu\rho\lambda}H^{\nu\rho\lambda} + 9\Gamma^{\rho\lambda}H_{\mu\rho\lambda})\epsilon^1 - \partial_\mu \epsilon^2 - \frac{1}{2}\Gamma^{\nu\rho}\partial_\nu h_{\mu\rho}\epsilon^2 + \frac{i}{4\cdot 5!}\Gamma^{\rho_1\cdots\rho_5}\Gamma_\mu \epsilon^2 F_{\rho_1\cdots\rho_5}\,,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\delta} h_{\alpha\mu} &= -i\bar{\epsilon^1}\Gamma_{(\alpha}\eta_{\mu)} + i\bar{\epsilon^2}\Gamma_{(\alpha}\eta^c_{\mu)}\,, \\
\tilde{\delta} A_{\alpha\beta\mu\nu} &= \bar{\epsilon^1}\Gamma_{[\alpha\beta\mu}\eta_{\nu]} + \bar{\epsilon^2}\Gamma_{[\alpha\beta\mu}\eta^c_{\nu]}\,, \\
\tilde{\delta}\eta^c_\mu &= \partial_\mu\bar{\epsilon^1} - \frac{1}{2}\bar{\epsilon^1}\Gamma^{\nu\rho}\partial_\nu h_{\mu\rho} + \frac{i}{4\cdot 5!}\bar{\epsilon^1}\Gamma_\mu\Gamma^{\rho_1\cdots\rho_5}F_{\rho_1\cdots\rho_5}\,, \\
\tilde{\delta} B^c_{\mu\nu} &= -\bar{\epsilon^2}\Gamma_{\mu\nu}\lambda^c - 2i\bar{\epsilon^1}\Gamma_{[\mu}\eta^c_{\nu]}\,, \\
\tilde{\delta}\Phi^c &= -\bar{\epsilon^1}\lambda^c\,.
\end{aligned} \tag{3.32}
$$

These transformations agree with the linearized local SUSY transformations of IIB supergravity under the identification that $\epsilon^1 = \epsilon^*$ and $\epsilon^2 = \epsilon$ [23].[4]

The commutator of two local supersymmetry transformations supposed to give all six types of local symmetry transformations:

$$
[\tilde{\delta}(\epsilon_1), \tilde{\delta}(\epsilon_2)] = i\tilde{\delta}(x) + i\tilde{\delta}(l) + i\tilde{\delta}(\epsilon) + i\tilde{\delta}(\Lambda.) + i\tilde{\delta}(\Lambda...) + i\tilde{\delta}(\Sigma)\,, \tag{3.33}
$$

where we have listed the general coordinate $\tilde{\delta}(x)$, local Lorentz $\tilde{\delta}(l)$, local supersymmetry $\tilde{\delta}(\epsilon)$, gauge transformations for the second $\tilde{\delta}(\Lambda.)$ and the fourth rank tensor $\tilde{\delta}(\Lambda...)$ and U(1) transformation $\tilde{\delta}(\Sigma)$, respectively. We can check eq. (3.33) at the linearized level with the transformations in eq. (3.32) as follows:

general-coordinate transformation with $x^\mu = \bar{\epsilon}_2^*\Gamma^\mu\epsilon_1 + \bar{\epsilon}_2\Gamma^\mu\epsilon_1^*$;

gauge transformation for $B.$ with $\Lambda_\mu = 2\bar{\epsilon_2}\Gamma_\mu\epsilon_1$;

gauge transformation for $A...$ with $\Lambda_{\alpha\beta\gamma} = (1/4i)\bar{\epsilon_2}^*\Gamma_{\alpha\beta\gamma}\epsilon_1 - (1/4i)\bar{\epsilon_2}\Gamma_{\alpha\beta\gamma}\epsilon_1^*$.

4. Conclusions and discussions

We have investigated the Wilson lines which couple to the supergravity multiplet in IIB matrix model. These fields may be introduced as the sources which couple to the Wilson lines. They transform in a definite way under the local supersymmetry transformation. The local supersymmetry is realized in matrix models if the perturbed theory is invariant under such a transformation. We have shown that it is the case at the linearized level of the symmetry for a class of massless modes. This conclusion follows from $\mathcal{N} = 2$ supersymmetry of IIB matrix model.

In matrix models, there is no distinction between NS-NS and R-R fields. For example the graviton $h_{\mu\nu}$ is a NS-NS field while the fourth rank antisymmetric tensor $A_{\mu\nu\rho\sigma}$ is a R-R field. They both appear in eq. (3.21). In the case of the complex fields, there are two different vertex operators for each complex field. In the case of $B_{\mu\nu}$ field, a T-duality argument suggests that the R-R field couples to the Wilson line which contains the single power of $[A_\mu, A_\nu]$ [32]. On the other hand it also couples to the NS-NS field as it was recalled in section 2. So we may interpret that $B_{\mu\nu}$ which couples to $[A_\mu, A_\nu]$ is the linear combination of NS-NS and R-R field as $B_{\rm NS} + iB_{\rm R}$. With such an interpretation, the one-loop effective action eq. (2.4) implies that $B^c_{\mu\nu}$ which couples to the third power of $[A_\mu, A_\nu]$ is of $B_{\rm NS} - iB_{\rm R}$ type. Presumably the situation is analogous for other complex fields.

[4] We remark that our metric $\eta^{\mu\nu}$ is of the opposite sign of that in [23]. We also recall that $\bar{\epsilon} \equiv \epsilon\Gamma^0$ in our conventions. After rescaling the gravitino field $\eta_\mu \to 2\eta_\mu$, our SUGRA transformations coincide with their $\kappa = 2$ case.

Since we have relied on the low energy approximation in this paper, our next goal is to understand the structure of the Wilson lines for massive modes of superstring. It looks likely that we have scratched the vertex operators for massive modes already in this paper. Therefore we may be able to understand them by further extending this work. Another possibly related problem is to understand the fully non-linear structure of the local SUSY transformations beyond the linearized approximation.

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A. SUSY transformation of graviton vertex operator

In this section, we investigate SUSY transformation of the Wilson lines which couple to a graviton and a four-th rank antisymmetric tensor. We show that the O(1) terms cancel and the result is O(k). We then discuss whether we can understand the result in terms of SUGRA transformations of known or unknown vertex operators. Although our physical understanding of it is still incomplete, we believe it is worthwhile to report our results for future investigations.

The SUSY transformation of eq. (3.21) gives rise to:

$$\begin{aligned}
&\mathrm{Str}\exp(ik\cdot A)\left\{i[A^\alpha,\bar{\epsilon}^1\Gamma^\beta\psi][A^\mu,A_\beta]+\frac{i}{2}\bar{\epsilon}^1\Gamma^{\alpha\beta\gamma}\psi[A^\mu,[A_\beta,A_\gamma]]+\right.\\
&\qquad +2\bar{\epsilon}^1\Gamma^\alpha\psi[k\cdot A,A^\beta][A^\mu,A_\beta]-\bar{\epsilon}^1 k\cdot\Gamma\psi[A^\alpha,A^\beta][A^\mu,A_\beta]-\\
&\qquad \left.-\frac{1}{4}\bar{\epsilon}^1\Gamma^{\alpha\beta\gamma}\psi[A^\mu,k\cdot A][A_\beta,A_\gamma]-\frac{1}{2}\bar{\epsilon}^1\Gamma^\beta\psi[A^\mu,k\cdot A][A_\beta,A^\alpha]\right\}h_{\alpha\mu}-\\
&-\mathrm{Str}\exp(ik\cdot A)\left\{\frac{i}{2}\bar{\epsilon}^2\Gamma^\alpha\psi[k\cdot A,A^\mu]h_{\alpha\mu}+\bar{\epsilon}^2\Gamma^{\rho\beta(\alpha}\psi[A^{\mu)},A_\beta]\partial_\rho h_{\alpha\mu}\right\}+\\
&+\frac{1}{2}\mathrm{Str}\exp(ik\cdot A)[A^\alpha,A^\beta][A^\mu,A^\nu]\bar{\epsilon}^1\Gamma^\rho\psi F_{\alpha\beta\mu\nu\rho}\,. \qquad \text{(A.1)}
\end{aligned}$$

For simplicity we have neglected cubic terms in ψ here.

In order to cancel the first line in the preceding expression which is O(1), we may modify the SUSY transformation eq. (1.2) as follows:

$$\delta^{(1)}\psi=\frac{i}{2}[A_\mu,A_\nu]\Gamma^{\mu\nu}\epsilon^1+ih_{\mu\alpha}P\exp(ik\cdot A)[A^\mu,A_\nu]\Gamma^{\alpha\nu}\epsilon^1\,,\qquad \delta^{(1)}A_\mu=i\bar{\epsilon}^1\Gamma_\mu\psi\,,\quad \text{(A.2)}$$

where $P\exp(ik\cdot A)\hat{O}$ implies $\int_0^1 d\sigma\exp(ik\sigma\cdot A)\hat{O}\exp(ik(1-\sigma)\cdot A)$. From SUSY transformation of the action, we obtain

$$\begin{aligned}
&\mathrm{Str}\exp(ik\cdot A)\left\{-i[A^\alpha,\bar{\epsilon}^1\Gamma^\beta\psi][A^\mu,A_\beta]-\frac{i}{2}\bar{\epsilon}^1\Gamma^{\alpha\beta\gamma}\psi[A^\mu,[A_\beta,A_\gamma]]+\right.\\
&\qquad \left.+\bar{\epsilon}^1\Gamma^{\alpha\beta\gamma}\psi[k\cdot A,A_\gamma][A^\mu,A_\beta]-\bar{\epsilon}^1\Gamma^\alpha\psi[k\cdot A,A^\beta][A^\mu,A_\beta]\right\}h_{\alpha\mu}\,. \quad \text{(A.3)}
\end{aligned}$$

Alternatively we can argue that the above expression vanishes due to the equation of motion. By combing these two contributions, we obtain

$$\begin{aligned}
&- \operatorname{Str}\exp(ik\cdot A)\left\{\frac{i}{2}\bar{\epsilon}^2\Gamma^{\alpha}\psi[k\cdot A, A^{\mu}]h_{\alpha\mu} + \bar{\epsilon}^2\Gamma^{\rho\beta(\alpha}\psi[A^{\mu)}, A_{\beta}]\partial_{\rho}h_{\alpha\mu}\right\} + \\
&+ \operatorname{Str}\exp(ik\cdot A)\Big\{\bar{\epsilon}^1\Gamma^{\alpha\beta\gamma}\psi[k\cdot A, A_{\gamma}][A^{\mu}, A_{\beta}] + \\
&\qquad + \bar{\epsilon}^1\Gamma^{\alpha}\psi[k\cdot A, A^{\beta}][A^{\mu}, A_{\beta}] - \bar{\epsilon}^1 k\cdot\Gamma\psi[A^{\alpha}, A^{\beta}][A^{\mu}, A_{\beta}] - \\
&\qquad - \frac{1}{4}\bar{\epsilon}^1\Gamma^{\alpha\beta\gamma}\psi[A^{\mu}, k\cdot A][A_{\beta}, A_{\gamma}] - \frac{1}{2}\bar{\epsilon}^1\Gamma^{\beta}\psi[A^{\mu}, k\cdot A][A_{\beta}, A^{\alpha}]\Big\} h_{\alpha\mu} + \\
&+ \frac{1}{2}\operatorname{Str}\exp(ik\cdot A)[A^{\alpha}, A^{\beta}][A^{\mu}, A^{\nu}]\bar{\epsilon}^1\Gamma^{\rho}\psi F_{\alpha\beta\mu\nu\rho}\,. \qquad (A.4)
\end{aligned}$$

We first focus on the terms which depend on ϵ^2 in the first line of (A.4). These kind of terms also originate from the gravitino coupling in eq. (3.16) after postulating the local SUSY transformation of η_μ as

$$\tilde{\delta}\eta_{\mu} = -\partial_{\mu}\epsilon^2 - \frac{1}{2}\Gamma^{\nu\rho}\partial_{\nu}h_{\mu\rho}\epsilon^2\,. \qquad (A.5)$$

The above expression can be obtained by expanding the covariant derivative $D_\mu\epsilon^2$ in $h_{\alpha\mu}$ which is identified with zehnvein. After this procedure however, we are still left with the following term

$$-\frac{i}{2}\operatorname{Str}\exp(ik\cdot A)\bar{\psi}\Gamma^{\alpha\nu\rho}[A^{\mu}, A_{\rho}]k_{\nu}\epsilon^2 h_{\alpha\mu}\,. \qquad (A.6)$$

We may try to obtain the remaining ϵ^1 dependent terms from (3.24) by postulating the following SUGRA transformation of anti-gravitino which is complex conjugate to eq. (A.5)

$$\tilde{\delta}\bar{\eta}^c_{\mu} = \partial_{\mu}\bar{\epsilon}^1 - \frac{1}{2}\bar{\epsilon}^1\Gamma^{\nu\rho}\partial_{\nu}h_{\mu\rho}\,. \qquad (A.7)$$

Although we can cancel or account for half of the numerical coefficients of a few terms in (A.4), we also introduce new types of terms in this procedure. We record the remaining discrepancy in what follows

$$\begin{aligned}
&\frac{1}{2}\operatorname{Str}\exp(ik\cdot A)\Big\{\frac{1}{2}\bar{\epsilon}^1 k_{\nu}\Gamma^{\alpha\beta\gamma\nu\rho}\psi[A^{\mu}, A_{\alpha}][A_{\beta}, A_{\gamma}] + \\
&\qquad + \bar{\epsilon}^1 k_{\nu}\Gamma^{\nu\rho\beta}\psi[A^{\mu}, A_{\alpha}][A^{\alpha}, A_{\beta}] + \bar{\epsilon}^1 k_{\nu}\Gamma^{\rho\beta\gamma}\psi[A_{\nu}, A_{\gamma}][A^{\mu}, A_{\beta}] + \\
&\qquad + \bar{\epsilon}^1 k_{\nu}\Gamma^{\rho}\psi[A_{\nu}, A_{\beta}][A^{\mu}, A^{\beta}] - \bar{\epsilon}^1 k_{\nu}\Gamma^{\nu}\psi[A^{\rho}, A_{\beta}][A^{\mu}, A^{\beta}] - \\
&\qquad - \bar{\epsilon}^1 k_{\nu}\Gamma^{\beta}\psi[A^{\mu}, A_{\nu}][A_{\beta}, A^{\rho}]\Big\} h_{\mu\rho}\,. \qquad (A.8)
\end{aligned}$$

The last term of eq. (A.4) is not inconsistent with the following SUGRA transformation of the anti-gravitino:

$$\tilde{\delta}\bar{\eta}^c_{\mu} = \frac{i}{4\cdot 5!}\bar{\epsilon}^1\Gamma^{\mu}\Gamma^{\rho_1\cdots\rho_5}F_{\rho_1\cdots\rho_5}\,. \qquad (A.9)$$

Although we can account for one half of it in this manner, we also obtain other terms with different tensor structures.

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